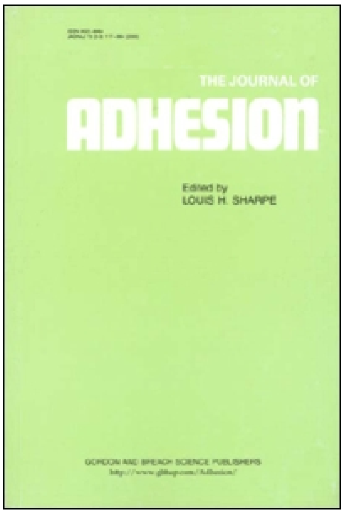

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Viscoelastic Analysis of an Adhesive Tubular Joint

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The investigations so far available with regard to stress analysis of adhesive joints assume that the adhesive is elastic. In the present analysis the time dependent properties of the adhesive are taken into account by assuming that the adhesive is viscoelastic. The viscoelastic analysis of a tubular joint has been attempted using a prony series fitting for the relaxation modulus of two adhesives. The long term redistribution of the stresses in the adhesive is evaluated using the finite element method.

INTRODUCTION

There has been extensive literature in connection with the elastic analysis of adhesive joints and this is evident from exhaustive list of references given in some of the reviews and monographs on adhesive joints.[1-5] Both classical solutions and recently computer methods, namely, finite difference and finite element techniques[6-8] have been employed for stress analysis of adhesive joints. All these analyses assume that the adhesive and the adherends are elastic. But it should be noted that in most of the adhesive joints, the adhesive is a viscoeleastic polymer and the material behaviour is time-dependent. The stresses and strains in the adhesive layer may vary with time depending upon the creep and relaxation properties of the adhesive material.

The present paper is directed towards the determination of the influence of the viscoelastic properties of the adhesive on the adhesive stresses, leading to a clear picture of the long term redistribution of stresses in the adhesive.

Viscoelasticity is a field which is well developed and a great impetus to this development has been the space applications like solid propellant grain analysis.

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The linear viscoelastic analysis is essentially based on the elastic-viscoelastic analogy,[9] where, by the application of Laplace transform, the time-dependent viscoelastic equations are converted to corresponding elastic field equations for which the solution procedures are well established. In essence, this method consists of replacing the elastic constants and load parameters in the expressions for elastic stresses and strains by the transformed modulii and loads and then taking inverse Laplace transforms, resulting in time-dependent viscoelastic stresses and strains. But the procedure becomes more and more difficult if the viscoelastic material representation has to be quite realistic. In other words, instead of a single Kelvin or Maxwell model representation, it is preferable to use generalized Kelvin or Maxwell models with a large number of elements if a realistic representation of creep and relaxation properties is desired over several decades of time. This in turn makes the evaluation of the inverse Laplace transforms a complicated task.

Another aspect of the problem is the solution procedure. Most of the classical solutions for stresses in adhesive joints are based on the assumption that the stress distribution is constant across the thickness of the adhesive. In reality it has been shown by Alwar and Nagaraja[8] using finite element technique that this assumption is not true in adhesive butt joints and a similar situation exists at the edges of tubular lap joints.[11] To obtain a real picture of the stresses and also if one has to take into account the complicated geometries of the adherend-adhesive combinations, the finite element method forms a very useful tool. The finite element technique can be directly used for viscoelastic analysis using a time-wise integration procedure.[12] But in the present analysis finite element technique is combined with an approximate inversion procedure suggested by Schapery[10] and the viscoelastic stresses in the adhesive for a tubular lap joint are analysed. This procedure of combining the finite element technique and the approximate inversion technique has been applied by Alwar and Pattabiraman in connection with solid propellant grain analysis[13] and recently by Adey and Brebbia[14] for nuclear reactor pressure vessels.

THE FINITE ELEMENT ANALYSIS

The tubular joint analysed is shown in Figure 1. The joint is similar to the one analysed by Lubkin and Reissner[15] and the various parameters used are indicated in the figure. The two adherends are similar and their Young's modulii and Poisson's ratios are 2.0×10^6 kg/cm^2 and 0.3 respectively, corresponding to steel. The properties used with regard to the viscoelastic adhesive are those corresponding to (a) Araldite GY 254 with Hardner XB 2606 and M/s CIBA and (b) Polymethylmethacrylate, PMMA. The CIBA adhesive is an epoxy-filler combination of considerable structural strength and durability especially suited for bonded joints with stress transmitting characteristics. The experimentally evaluated creep curve for this epoxy is given in Ref. 16 and has been used to determine the relaxation modulus. The relaxation modulus, $E_{rel}(t)$ can be taken approximately equal to the inverse of $\sigma_0/\varepsilon(t)$ which is the creep compliance, $J(t)$. Hence from the creep data supplied the variation of $E_{rel}(t)$ with time t, is obtained. In order to find out the operational modulus $E(p)$ to be used in the viscoelastic analysis, Schapery's method[10] of fitting a Dirichlet or Prony's series by collocation with the experimental data has been employed and by this way, a realistic viscoelastic material representation is made possible.

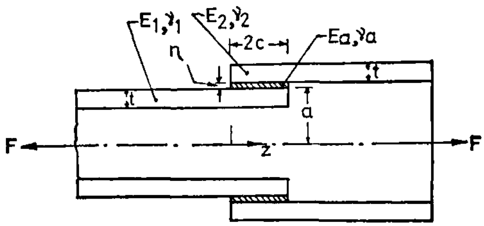

FIGURE 1 Tubular joint analysed.

The relaxation modulus is chosen in the following form

$$E_{rel}(t) = E_e + \sum_{k=1}^{n} E_k \exp(-t/\tau_k) \tag{1}$$

where E_e is the equilibrium modulus, τ_k are relaxation times, and E_k are constants.

This series is collocated with the experimental relaxation modulus curve for the material to determine the constants E_k and the relaxation times τ_k. The collocation procedure is as follows: first N decades of time intervals are selected over the transition range of the experimental curve and one value of τ_k is chosen in each of these N decades of time. The next step is to collocate the series at N points on the experimental curve. A set of N simultaneous equations are obtained involving the N unknown constants $E_1, E_2, \ldots E_N$. Solving these equations the series is completely determined. The experimental curve and the prony's series fitting is shown in Figure 2a for the CIBA adhesive and in Figure 2b for PMMA and it is seen that there is close agreement.

The advantage of the prony's series is that its Laplace transform is easily obtained. The transform is,

$$E_{rel}(p) = \frac{E_e}{p} + \sum_{k=1}^{n} \frac{E_k}{p+1/\tau_k} \tag{2}$$

The transformed modulus $E(p)$ is given by

$$E(p) = pE_{rel}(p) = E_e + \sum_{k=1}^{n} \frac{p\tau_k E_k}{1+p\tau_k} \tag{3}$$

Employing Eq. (3) the operational modulus $E(p)$ can be obtained and the variation of $E(p)$ with t is shown in Figure 3a for the CIBA adhesive and in Figure 3b for PMMA.

This transformed modulus $E(p)$ is substituted for E in the associated elastic solution.

To obtain the transform inversion Schapery's direct method[10] is used. This method states that if $\phi(t)$ is the viscoelastic response and $\phi(p)$ is the

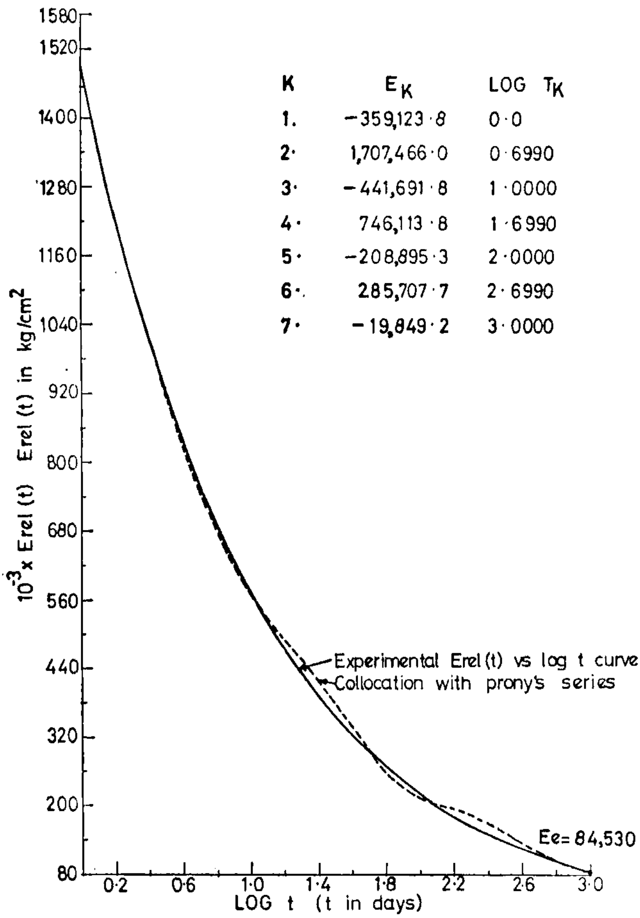

FIGURE 2a Comparison of experimental and Prony's series fitting of $E_{rel}(t)$ Vs log t for CIBA adhesive.

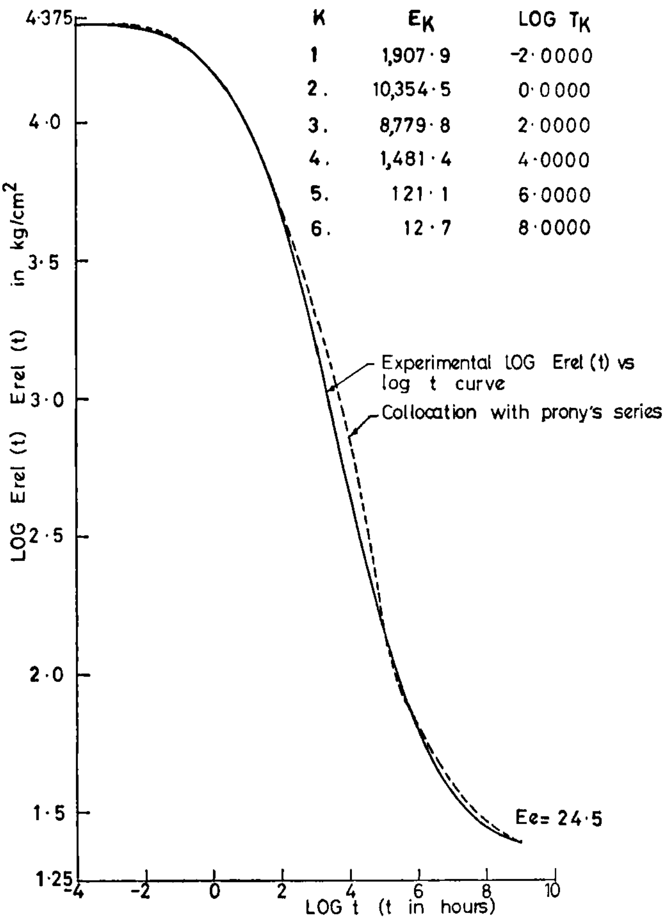

FIGURE 2b Comparison of experimental and Prony's series fitting of log $E_{rel}(t)$ *Vs* log t for PMMA.

Laplace transform of $\phi(t)$ whose values are known for all real non-negative values of the transform parameter 'p', then

$$\phi(t) = [p\phi(p)]_{p=0.5/t}$$

If the expressions for the transformed stresses, strains etc., are obtained analytically the subsequent inversion of these expressions is achieved by multiplying them by the transform parameter p, and obtaining the value of the expressions at $p = 0.5/t$. However, if numerical procedures such as the finite element method are used, then the inversion is done by substituting $p = 0.5/t$ for discrete values of t in the quantities involving the transformed parameters like $E(p)$, $v(p)$, etc., and the transformed load parameters.

In the finite element technique employed here linearly-varying-strain triangular elements have been used. The results obtained are the stresses at the centroid of each element and because of this the plotted results are at a level about one-tenth the thickness of the adhesive from the adherend-adhesive interfaces, and a little away from the edges of the joint.

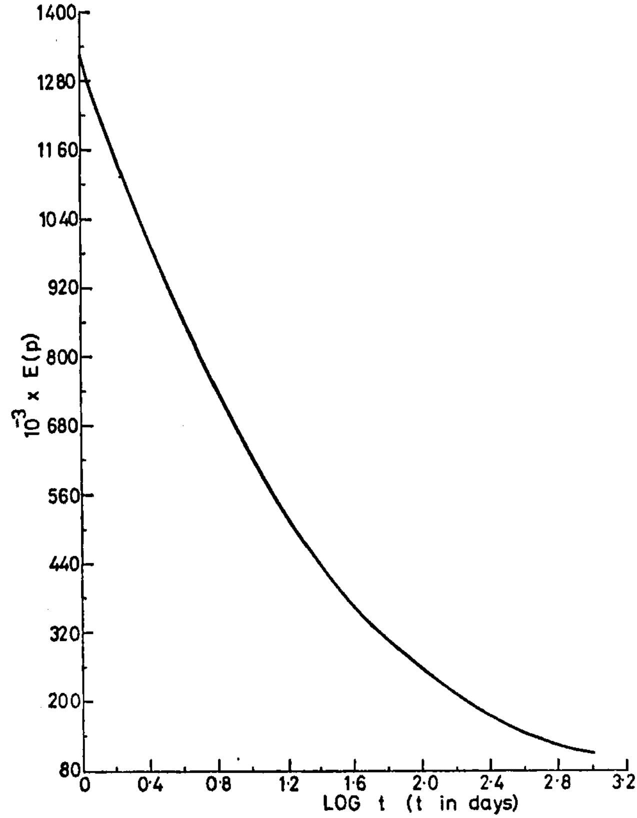

FIGURE 3a *E(p) Vs* log *t* for CIBA adhesive.

Figure 4 shows the finite element configuration for the tubular joint with the various parameters and boundary conditions. Results are shown at two levels, level 1, one-tenth of the adhesive thickness from the inner adherend-adhesive interface and level 2, a similar distance from the outer adherend-adhesive interface.

The programme used has been specially developed by the authors to study problems of adhesive joints of various types.[8, 11] Each solution, performed on the IBM 370/155 System, took 38 seconds of CPU time.

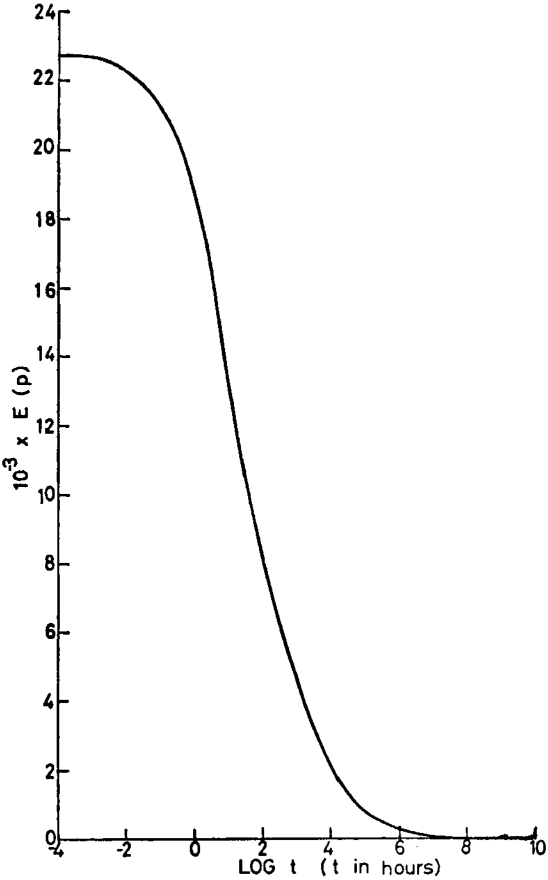

FIGURE 3b *E(p) Vs* log *t* for PMMA.

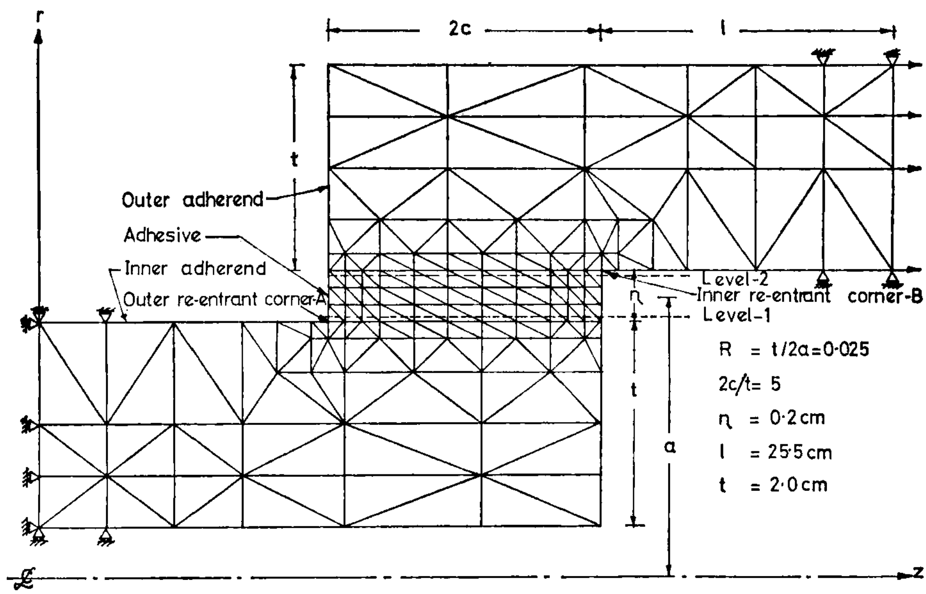

FIGURE 4 Finite element configuration (not to scale).

RESULTS AND DISCUSSION

In order to check the suitability of the finite element method the results obtained for the elastic analysis of a tubular lap joint from the finite element technique are compared with those of Lubkin and Reissner.[15] Figure 5 shows the normal stress concentration factor N, and Figure 6, the shear stress concentration factor T. It can be seen that the agreement is quite good.

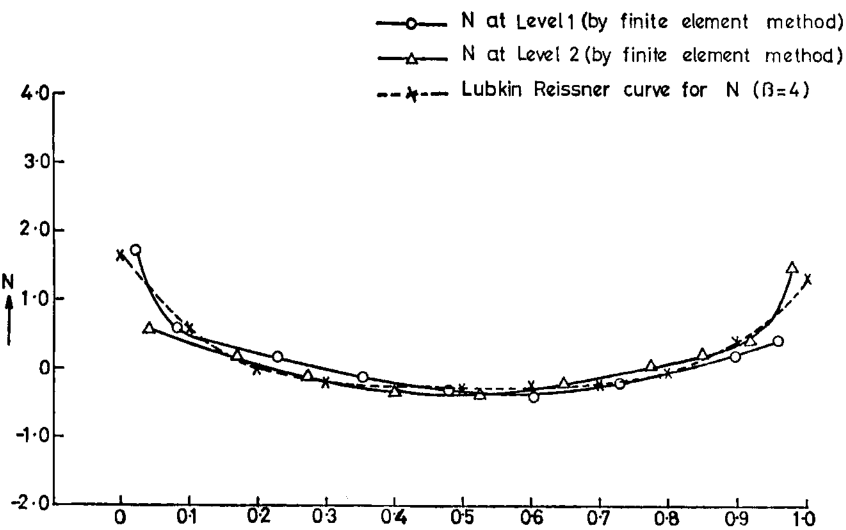

FIGURE 5 Comparison of finite element method results with those from Lubkin-Reissner theory.

Figures 7 and 8 show the variation of the normal stress concentration factor N for the CIBA adhesive along the overlap of the joint for four different values of time, t. The stress concentration factor is shown at level 1 in Figure 7 and at level 2 in Figure 8. The variation of time is from $t = 1$ day to $t = 1{,}000$ days.

The figures indicate the reduction of stress with time and that the stress at the outer reentrant corner A (Figure 4) is always higher than that at the inner reentrant corner B. At level 1, N is quite high being 5.80 near the outer reentrant corner at the beginning ($t = 1$ day). It reduces to 2.49 at $t = 1000$ days, a reduction of nearly 57%. At the same level, at the other end of the overlap the stress is quite small. The normal stress is tensile at the ends of the overlap with a region of compressive stress over most of the overlap.

At level 2, a similar trend is seen with the magnitude of N, smaller than at level 1.

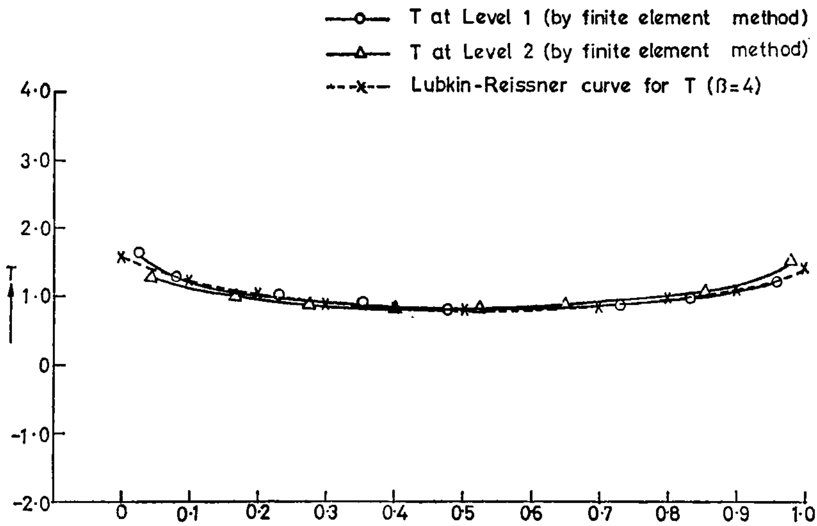

FIGURE 6 Comparison of finite element method results with those from Lubkin-Reissner theory.

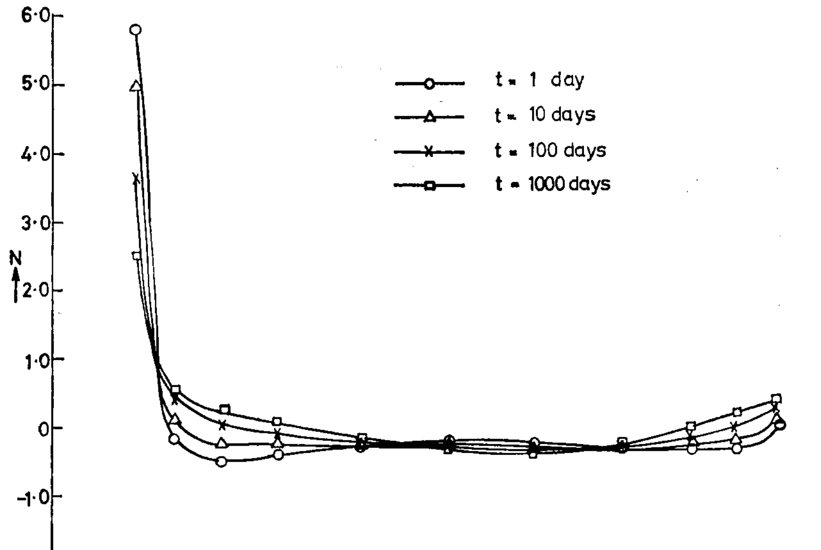

FIGURE 7 Variation of "N" along overlap at level 1 (CIBA adhesive).

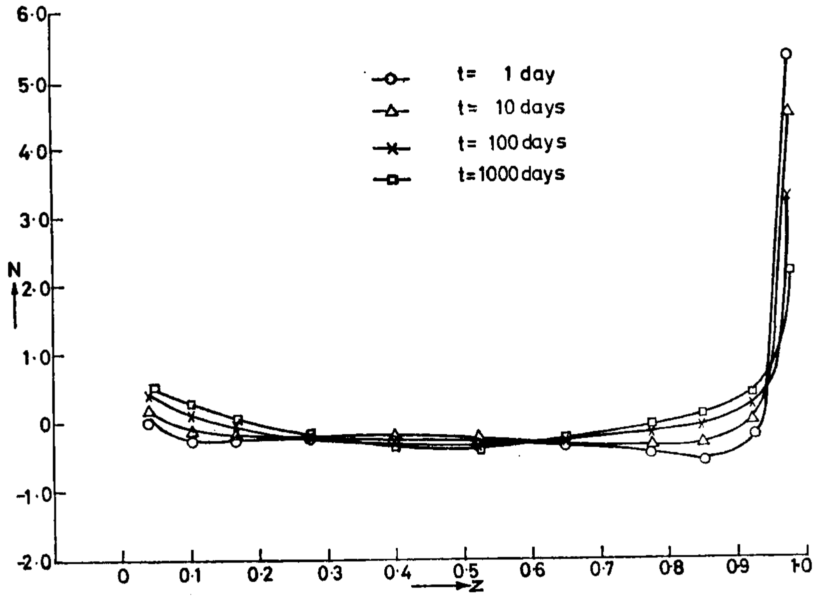

FIGURE 8 Variation of "N" along overlap at level 2 (CIBA adhesive).

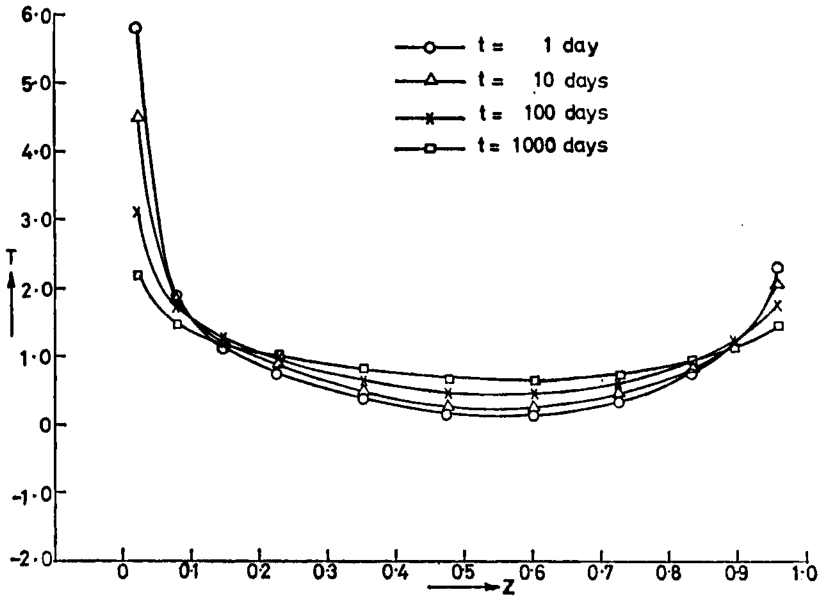

FIGURE 9 Variation of "T" along overlap at level 1 (CIBA adhesive).

It is to be noted here, that for the CIBA adhesive which is very stiff, the values of β are very small. According to Bikerman[3] the Lubkin–Reissner theory is not applicable for values of β less than 4 and hence the trend exhibited in Figures 7 and 8 is not similar to what may be expected from Lubkin–Reissner theory.

Figures 9 and 10 show the variation of the shear stress concentration factor T along the overlap for the CIBA adhesive. Figure 9 shows the variation at level 1 and Figure 10 that at level 2. Once again the maximum values are found at the outer reentrant corner. Near A, at level 1, T is 5.81 for $t = 1$ day reducing to 2.18 for $t = 1000$ days with a reduction of about 62%. The stress concentration at the other end of the overlap varies from 2.31 at $t = 1$ day to 1.45 at $t = 1000$ days with 37% reduction.

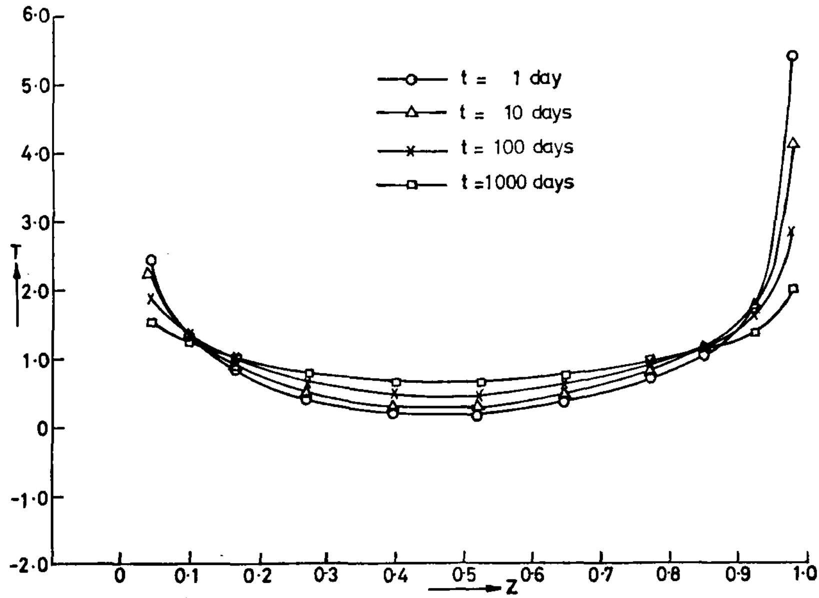

FIGURE 10 Variation of "T" along overlap at level 2 (CIBA adhesive).

At level 2 the same trend is evident, the concentration now being at the inner reentrant corner B. The magnitude varies from 5.41 for $t = 1$ day to 2.01 for $t = 1000$ days, a reduction of nearly 62%. At the other end of the overlap T reduces from 2.47 to 1.54 over the same time range as above, with a 37% reduction.

Figures 11 and 12 show the variation of N and T along the overlap length at level 1 for PMMA. Figure 11 shows the variation of N and Figure 12, that of T. A trend similar to that of the previous case, i.e. reduction of stress with time is seen, the maximum values being at the outer reentrant corner A.

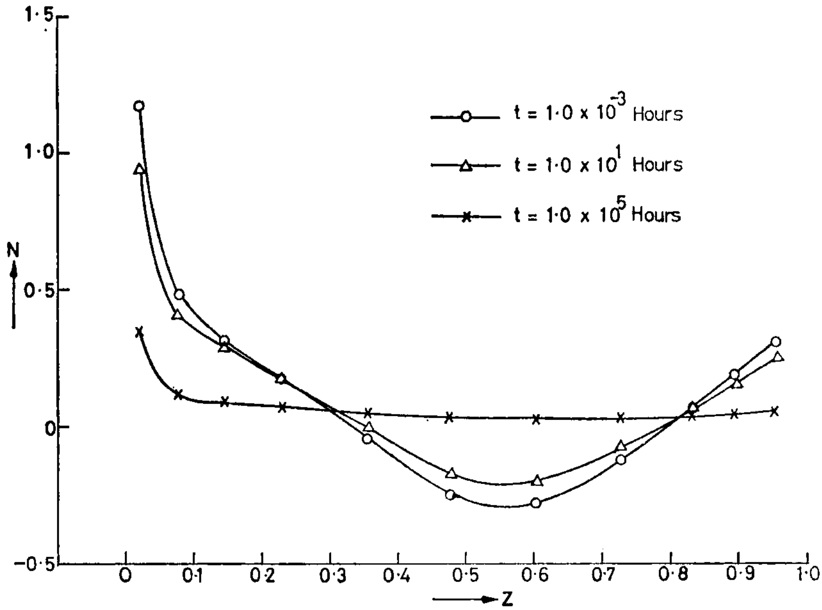

FIGURE 11 Variation of "N" along overlap at level 1 (PMMA).

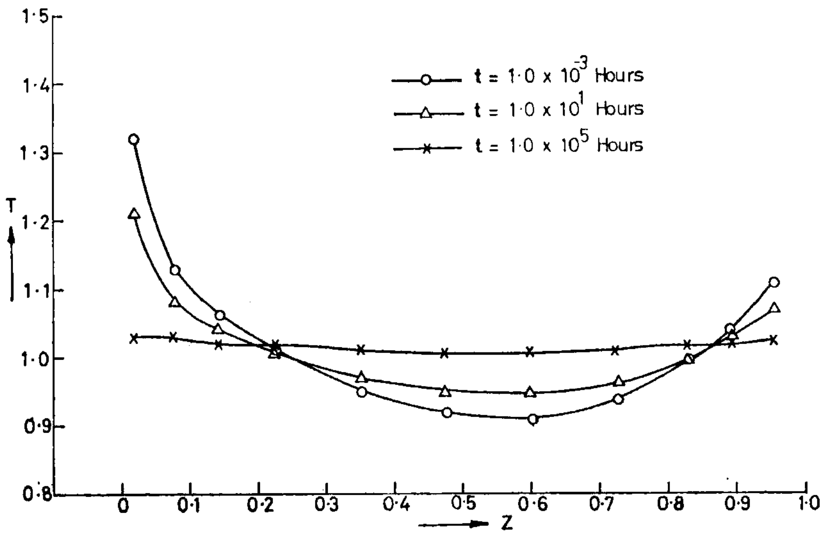

FIGURE 12 Variation of "T" along overlap at level 1 (PMMA).

The value of N reduces from 1.18 at $t = 1.0 \times 10^{-3}$ hours to 0.15 at $t = 1.0 \times 10^{5}$ hours, a reduction of 87% over 8 decades of time. At the other end of the overlap the same trend is seen with magnitudes that are quite small.

Figure 12 shows the variation of shear stress concentration T with time, at level 1. The maximum value of T is once again found at the outer reentrant corner A. T reduces from 1.32 to 1.03 over the eight decades of time, a reduction of 22%.

At level 2 the magnitudes of the stress concentrations N and T are lower than those at level 1 and the maximum values occur at the inner reentrant corner B.

The foregoing analysis and conclusions are valid for long term behaviour, which information is useful in designing many of the conventional structural adhesive joints. The method of analysis presented here is also applicable to short time behaviour as employed in Ref. 13, for example, within a period of the order of a few milliseconds, which might be of interest in solid propellant rocket design specifically from the point of view of the interdependence between strain rate and failure criteria.

SUMMARY

Viscoelastic analysis of an adhesive tubular joint has been performed for the first time, using the finite element method with a prony series fitting for the relaxation modulus of the adhesive. For a typical epoxy it has been found that not only the elastic stresses are different at different levels but also the viscoelastic response shows considerable variation from one level to another. As large a reduction as 57% is noticed in the normal stress and an even larger reduction of 62% is noticed in the shear stress over three decades of time.

The technique used in this report for the solution of the long term response can be employed for a study of the short-term response also.

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Nomenclature

E_1	Young's modulus of the inner adherend
E_2	Young's modulus of the outer adherend
E_a	Young's modulus of the adhesive
E_e	equilibrium modulus of the adhesive
E_k	constants in the prony series
$E(p)$	operational modulus
$E_{rel}(t)$	relaxation modulus of the adhesive
$E_{rel}(p)$	laplace transform of $E_{rel}(t)$
F	force applied on the joint
N	normal stress concentration factor ($= \sigma/\tau_m$)
R	relative tube thickness ($= t/2a$)
T	shear stress concentration factor ($= \tau/\tau_m$)
a	radius of the middle layer of the adhesive
$2c$	length of the overlap
p	the laplace parameter
t	thickness of the adherends, and time in viscoelastic analysis
z	distance from the loaded end of the inner adherend
β	flexibility of the joint ($= \eta E/tE_a$)
η	thickness of the adhesive
ν	Poisson's ratio of the adherends
ν_a	Poisson's ratio of the adhesive
σ	normal stress in the adhesive
τ	shear stress in the adhesive
τ_k	relaxation times
τ_m	mean shear stress ($= F/4\pi ac$)
$\phi(t)$	a time-dependent function
$\phi(p)$	laplace transform of $\phi(t)$